

# A.P. Calculus Summer Review

Calculus is a difficult course, not as much because of the new material, but more because it requires knowing everything from previous courses. It will require you to utilize algebra within the process of solving calculus equations. To do well in this class, you should expect to devote an hour a day (including weekends) to studying and/or reviewing each day of the school year. If you are truly prepared for this class, this will be sufficient. If your Pre-Calculus skills are lacking however, you will need to devote even more time. If you are too dependent on your calculator to do your work, you will find your Calculus tests to be quite trying. This review is designed for you to get a head start on the year. This way we can hit the year rolling. We will not have time to devote to reviewing skills you should have already mastered.

Face this challenge just like you would in any sport, music, art, debate, useless trivia, or any other competition. Calculus is like a competition; homework is practice, tests are games, and the AP Calculus Test is the state meet. Just like many high school sports have summer programs so that they can succeed at their state competition; here is your Calculus summer program.

There will be a test on day number 2 next year covering this review. This is also your first Calculus Assignment so be sure to show all of your work. This Review will tell me how committed you are to learning Calculus by reviewing over the summer. Do not wait until the day before school starts to begin this work. You will be sorry. You may not have a text book, but there are numerous resources available on the Web. You can also reach me for hints/help via email at:

[msturtevant@lwsd.org](mailto:msturtevant@lwsd.org)

## 1. Factor out the greatest common term in each case (and simplify if possible):

1.  $3x^4 + 4x^3 - x^2$       2.  $2\sqrt{x} + 6(x)^{3/2}$       3.  $4(x+1)^2 - (x^2 - 1) + (3x^3 + x^2 - 2x)$

## 2. Factoring Quadratic Expressions (Factor completely as a product of 2 or more binomial expressions)

1)  $x^2 - 3x + 2$

6)  $e^{2x} + 2 + e^{-2x}$

2)  $x^2 - 9$

7)  $x^4 - 7x^2 + 12$

3)  $x^2 + 5x - 6$

8)  $\cos^2 x + 6\cos x + 8$

4)  $x^2 + 5x + 6$

9)  $1 - \sin^2 x$

5)  $2x^2 + 5x - 3$

10)  $-10\cos^2 x - 3\sin x + 9$

## 3. Simplification through Cancellation (Reduce each of the following fractions to lowest terms)

1)  $\frac{3x+9}{6x}$

5)  $\frac{x^{1/2} - x^{1/3}}{x^{1/6}}$

2)  $\frac{x^2}{\sqrt{x}}$

6)  $\frac{\sqrt{x-1} + (x-1)^{3/2}}{\sqrt{x-1}}$

3)  $\frac{(x+1)^3(x-2) + 3(x+1)^2}{(x+1)^4}$

7)  $\frac{1 - (\sin x + \cos x)^2}{2 \sin x}$

4)  $\frac{x^3 - 9x}{x^2 - 7x + 12}$

8)  $\frac{9 - x^{-2}}{3 + x^{-1}}$

#### 4. Simplification through combination

Rewrite each of the following in simplest form through the use of common denominators, rationalizing denominators, rationalizing numerators, etc.:

$$1) \frac{(x-1)(x+3) - (x+1)^2}{x+1}$$

$$2) \frac{\frac{2}{x+6} - \frac{1}{3}}{x}$$

$$3) \frac{-1}{\sqrt{2-x} + \sqrt{2}}$$

$$4) \frac{1}{(8x^3)^{1/3}} (4x^2)^{1/2}$$

$$5) \frac{x - \frac{1}{y}}{\frac{x}{y}}$$

$$6) \frac{1}{x+1} - \frac{1}{x-1} - \frac{2}{x^2-1}$$

$$7) -\frac{x^2}{\sqrt{1-x^2}} + \sqrt{1-x^2} + \frac{1}{\sqrt{1-x^2}}$$

**5. Trigonometry** (For each of the following, combine as a single trig function. If not possible, rewrite in terms of  $\sin x$  and  $\cos x$ )

$$1) (\cos 4x)(\cos x) - (\sin 4x)(\sin x)$$

$$2) \sin^2 x + \cos 2x$$

$$3) \csc x - \cot x$$

$$4) (\csc x + \cot x)(1 - \cos x)$$

$$5) \sec^4 x - \tan^4 x - 2 \tan^2 x$$

**6. Solving** (Solve for  $x$  without your calculator:)

$$1) x^2 - x = 6$$

$$2) x^2 + 7x + 6 = 0$$

$$3) (x-1)(x+2) < 0$$

$$4) 2^x = 20$$

$$5) |2x+1| = x+3$$

$$6) |x-3| \leq 5$$

$$7) 5^{x+1} = 25$$

$$8) \log_3 x^2 = 2 \log_3 4 - 3 \log_3 2$$

$$9) 8x^3 + 27 = 0$$

$$10) 12x^3 - 23x^2 - 3x + 2 = 0$$

$$11) 2x + 1 = \frac{5}{x+2}$$

**7. Evaluating** (Simplify whenever possible) – all should be done without any technology

$$1) \sin^2\left(\frac{\pi}{4}\right) - \cos^2\left(\frac{\pi}{4}\right)$$

$$2) 3 \sin\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{3}\right)$$

$$3) f(x) = 3x^4 - 7x^2 + 2x - 9, \text{ find } f(-x)$$

$$4) f(y) = 3(y-1)^2 - 4y + 7, \text{ find } f(2x+1)$$

$$5) f(x) = x^2 + 2x + 5, \text{ find } f(x+h)$$

$$6) f(x) = 2x + 3, \text{ find } \frac{f(x+h) - f(x)}{h}$$

$$7) f(x) = x^2 - 4x + 2, \text{ find } \frac{f(x+h) - f(x)}{h}$$

## 8. Miscellaneous

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- 1) Distance between (3, 7) and (6, 9)
- 2) Equation of a line through (2, 2) and (3, 6)
- 3) Slope of a line perpendicular to  $y = 2x - 3$
- 4) Point of Intersection between:  $x - y = 3$                        $x + y = 1$
- 5) Points of Intersection between:  $y = x^2$                                        $y = -2x + 3$
- 6) What are the domain and range of the function:  $f(x) = \sqrt{x - 4} + 2$
- 7) What is the domain of the function?  $g(x) = \frac{3x+1}{\sqrt{x^2+x-2}}$
- 8) What are the ranges of the following functions?
  - a.  $f(x) = \cos^{-1} x$
  - b.  $g(x) = \tan^{-1} x$
  - c.  $h(x) = \sec^{-1} x$
- 9) Find the inverses of:
  - a.  $f(x) = 2x + 3$
  - b.  $g(x) = x^2 + 3$ ; restrict the domain of  $g(x)$  to  $x \leq 0$
  - c.  $h(x) = 3(2)^{x-4}$
  - d.  $j(x) = 2 \sin(3x - 4) + 5$

## 9. Graphing

(Sketch graphs of each of the following without use of a calculator. Use your calculator to confirm your graph)

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- 1)  $2x - 3y = 6$
- 2)  $y = x^2$
- 3)  $y = \frac{1}{x}$
- 4)  $y = \sqrt{x}$
- 5)  $y = 2^x$
- 6)  $y = \text{Log } x$
- 7)  $y = |x|$
- 8)  $y = \left(\frac{1}{2}\right)^x$
- 9)  $y = (x^2 - 1)(x - 2)^2$
- 10)  $x^2 + y^2 = 16$
- 11)  $y = \frac{4x-5}{x-2}$
- 12)  $y = \frac{x(x-2)}{x-2}$
- 13)  $y = \cos x$
- 14)  $y = 3 \cos(2x)$
- 15)  $y = \sin\left(2x - \frac{\pi}{3}\right)$
- 16)  $y = \log(x - 5) + 2$
- 17)  $y = |x - 2| + 3$
- 18)  $y = (x + 3)^2 + 2$
- 19)  $y = \frac{3}{x-2} - 4$
- 20)  $y = \frac{(x+3)(x-1)}{(x+1)(x-2)}$
- 21)  $y = 2^{-x}$
- 22)  $f(x) = \begin{cases} 2x^2 - x & \text{if } x < 3 \\ 7 & \text{if } 3 \leq x \leq 5 \\ -\sqrt{x^2 - 25} + 7 & \text{if } x > 5 \end{cases}$